



Efficient automatic estimation of soil constitutive model parameters via particle swarm optimization

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Abstract

In this study, we automatically estimated the parameters of the modified Cam-Clay model, a representative constitutive model for soil. The estimation was carried out by minimizing the objective function using the dynamic multiswarm particle swarm optimization (DMS-PSO) algorithm, which is an improvement over the original PSO. The objective function was newly defined by quantifying the discrepancy between the targeted results and the model calculations in q - p' - v space. DMS-PSO divides particles into several islands to search globally and prevent local solutions, and even particles that fall into a local solution can be relocated. To evaluate the automatic estimation performance of DMS-PSO, we examined whether model parameters could be correctly estimated from the calculation results (Consideration (1)) and whether the DMS-PSO algorithm could consistently obtain the same parameter values when reproducing the experimental results (Consideration (2)). Regarding Consideration (1), the objective function was consistently smaller than 1.0×10^{-6} when the number of particles was greater than 400 and the number of islands was greater than 40. At this time, the parameter values could be estimated to the fifth decimal place. When two experiments were conducted, the estimation was obtained approximately 1.5 times faster than when only one was conducted. Regarding Consideration (2), the coefficient of variation of the parameters obtained from 100 estimations was at most 1%, and the parameter values were estimated to be approximately the same each time. In addition, narrowing the solution search range based on soil physical properties could reduce the variation in parameters by approximately 10%. Additionally, the parameters could be accurately estimated by data from at least two mechanical experiments.

Keywords Cam-Clay model · Data assimilation · Oedometer test · Particle swarm optimization · Triaxial compression test

1 Introduction

Many studies and designs apply the finite element method (FEM). In the FEM, the analysis results are mainly affected by how the parameters of the constitutive model are determined. Other methods are also affected by this constraint. Constitutive model parameters are often determined by comparing numerical simulation results with laboratory test results and by varying the parameters by trial and error so that the numerical simulations better fit the experimental

results. However, the skill of a researcher or a designer can affect the determination of constitutive parameters. Moreover, the optimal type and number of experiments are very useful information, but there is no objective basis on how to select experiments. In addition, many researchers do not evaluate the degree of experimental result reproduction quantitatively.

Many studies have proposed various algorithms to address this issue via inverse analysis. Thus, multiple algorithms have been used to address this issue, including genetic algorithms [10] (GAs), particle swarm optimization [15] (PSO), and particle filtering [5] (PF).

Pal et al. [24], Samarajiva et al. [27], and Rokonzuzman and Sakai [25] estimated parameters for high experimental and analytical compatibility using GAs based on the results of drained triaxial compression tests of sand. Furthermore, Yin et al. [11] developed the real coded genetic

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algorithm (RCGA) and reproduced the experimental results of several drained and undrained triaxial compression tests of sand via a constitutive model. Additionally, Song et al. [30] developed an improved algorithm based on the RCGA and used a similar approach to that used by Yin et al. [11]. Gras et al. [7] obtained parameters for constitutive models using a GA for triaxial compression and extension test results. Furthermore, reproductions of in situ pressure meter tests have been attempted (Levasseur, 2007) [17].

Regarding PSO, Yazdi et al. [39] estimated the parameters of constitutive models using PSO based on drained triaxial tests of sand, while Knabe et al. [16] estimated parameters from oedometer and drained triaxial compression tests of clay. Zhu et al. [43] estimated the parameters of constitutive models from drained/undrained triaxial compression tests and isotropic consolidation tests of clay using multiadaptive particle swarm optimization (MAPSO). Li and Zhu [18] estimated parameters that could effectively reproduce the results of repeated triaxial tests using PSO. Additionally, Zhang et al. [40, 41] utilized hybrid moving boundary particle swarm optimization (hmPSO) to estimate parameters capable of reproducing pressure meter test results using models of unsaturated soil. Furthermore, PSO has been applied to various geotechnical engineering problems, including parameter estimation of moisture characteristic curves [35], prediction of maximum surface settlement during tunnel excavation [9], and estimation of sliding surfaces in arc sliding analysis [13], as summarized by Hajihassani et al. [8].

Murakami et al. [20] and Shuku et al. [28] attempted parameter estimation from triaxial compression tests using PF and clarified that estimation is easier for undrained shear because more information is obtained. Yan et al. [38] estimated soil moisture states and soil hydraulic parameters using PF.

Furthermore, recent attempts have been made to reproduce experimental results using reinforcement learning. Zhang et al. [42] reproduced experimental results using a model trained on learning data of the response of constitutive models. Additionally, Wu et al. [37] created learning data based on the response of the discrete element method (DEM) and showed the advantages and disadvantages of long short-term memory (LSTM) and multilayer perceptron (MLP) models in reproducing experimental results.

These methods are classified as heuristic algorithms and have been applied to many problems. However, these algorithms have the disadvantage of being computationally expensive [12]. In heuristic algorithms, narrowing the solution search range significantly improves the calculation stability and reduces the computational cost. Zhang et al. [41] showed that a narrower search range is more appropriate when estimating the parameters of a constitutive model. However, in most previous studies, the impact of

the search range was not studied, but the estimation was conducted within a possible range of parameters for the soil. The number of soil samples obtained from the field are often limited, and therefore, determining which tests should be conducted on such few experimental samples is a critical problem. However, there are no studies that indicate the optimal type and number of tests for parameter estimation. In previous studies, parameters were estimated by using the error between the targeted and analytical results as the objective function in different spaces and at different scales according to the experiments conducted. To investigate the optimal tests for accurate parameter estimation, it is necessary to quantify errors between the targeted and calculation results within the same space and scale, such as for drained/undrained triaxial compression tests performed under different confining pressures and oedometer tests.

In addition, the performance of the algorithm should be verified when a new objective function is defined to estimate the parameters of a soil constitutive model. However, there are few studies that verify algorithms with a self-defined objective function because these algorithms have already been evaluated with a general benchmark function.

Because heuristic methods use random numbers, it is necessary to evaluate whether the same parameter values are obtained each time. Since a constitutive model has many parameters, there is a possibility to obtain several parameter sets with few experimental results, even though the degree of reproduction would be approximately the same in sight. Gras et al. [7] recommend conducting multiple estimations and using the parameters with the smallest objective function. In addition, many studies have been limited to discussing whether such parameters could be obtained or reproduce mechanical tests. Knabe et al. [16] used a sensitivity matrix, confidence intervals, and residual analysis to discuss the accuracy of the parameters obtained, but there are few such examples.

In this study, the parameters of the modified Cam-Clay model [26, 36], a representative soil constitutive model, were automatically estimated. The estimation was carried out by minimizing the objective function using the dynamic multiswarm PSO (DMS-PSO) algorithm [19], which is an improvement over the original PSO. The objective function was newly defined by quantifying the discrepancy between the targeted results and the model calculations in q - p' - v space, where v is the specific volume, p' is the mean effective stress, and q is the deviator stress. In critical state soil mechanics theory, which is the basic theory of soil constitutive models, the mechanical behavior of soil is described in q - p' - v space. In this space, the constitutive model can describe the mechanical behavior without distinction among tests such as drained/undrained triaxial tests and oedometer tests. Therefore, the objective

function defined in q - p' - v space can quantify the discrepancy regardless of the test conditions, which is one of the novelties of this study.

PSO is intuitive and easy to implement, and it can effectively solve highly complex and nonlinear optimization problems, as noted in several studies [8, 16, 18]. Therefore, in previous studies, PSO has been adopted for many geotechnical problem [8]. PSO has also been used to estimate the parameter values of constitutive models. Kaushik and Kumar (2016) [14] have indicated the superiority of PSO in some cases over GA, which also has many applications. A disadvantage of PSO is the possibility of falling into an incorrect local solution. Therefore, the DMS-PSO algorithm proposed by Liang and Suganthan [19] was adopted to avoid falling into local solutions. The details of DMS-PSO are described in Sect. 3. DMS-PSO can be used to search for solutions more efficiently and reduce the computational cost.

One of the most novel approaches of this study is the evaluation of the performance of the DMS-PSO algorithm to estimate parameters of the soil constitutive model in two ways.

- (1) The first approach considers if the constitutive model parameters can be correctly estimated from the calculation results (Consideration (1)).
- (2) The second approach considers if the DMS-PSO algorithm always obtains the same parameter values from experimental results (Consideration (2)).

Regarding Consideration (1), the results calculated by the constitutive model are regarded as experimental results, and the performance of the DMS-PSO algorithm can be examined. Commonly, a benchmark function is used to examine the performance. The performance of DMS-PSO has already been verified by Liang and Suganthan [19] using several benchmark functions. However, it is necessary to verify the performance of DMS-PSO when defining an objective function for estimating the parameter values of soil constitutive models. In this study, we investigated the verification of the DMS-PSO algorithm by attempting to estimate parameters with it 100 times.

Even if the DMS-PSO algorithm is verified by Consideration (1), it must also follow Consideration (2). Regarding Consideration (2), the estimation accuracy was also evaluated by examining the coefficients of variation of the parameters obtained by 100 parameter estimation attempts.

The effect of the search range on the time required for parameter estimation and the accuracy of the estimation was also investigated. To narrow the search range, we focused on physical properties such as the liquid limit and plastic index. For example, Skempton and Jones [29] proposed an equation to obtain the compression index from

the liquid limit; other studies have proposed similar equations [2, 4, 31, 32, 34]. This result can contribute to a reduction in computational cost.

In this study, we also examined which tests would make it easier to estimate the solution. Regarding Consideration (1), the type and number of experiments that are optimal to perform for parameter estimation were discussed, conducting undrained/drained triaxial compression tests and oedometer tests as examples. Regarding Consideration (2), parameters were estimated from triaxial compression tests or from triaxial compression tests and oedometer tests, and the effects of the type of experiments on the differences and variations in the parameter values obtained were discussed.

2 The target problem in this study

2.1 Calculation results and targeted parameters for the evaluation of DMS-PSO performance

In this study, we simulated the consolidated drained (CD) test, the consolidated undrained ($\overline{CU}$) test, and the oedometer (OED) test via the Cam-Clay model. Table 1 shows the correct parameters for alluvial clays, as determined by Nakai et al. [21]. Figures 1, 2 and 3 show the calculation results using the parameters shown in Table 1. The performance of the DMS-PSO algorithm was evaluated by examining whether the parameter values shown in Table 1 were obtained when the parameters were estimated from the results calculated with DMS-PSO algorithm.

2.2 The experimental results for the evaluation and validation of DMS-PSO

Figures 4 and 5 show the results of the $\overline{CU}$ test and the OED test conducted on a reconstituted specimen of Joetsu clay [22]. The reconstituted clay specimen was formed by a one-dimensional consolidation process after remolding and had an initial water content approximately twice the liquid limit.

Table 1 Targeted parameters in this study

Compression index	$\hat{\lambda}$	0.210
Swelling index	$\hat{\kappa}$	0.035
Critical state index	$\tilde{M}$	1.500
Intercept of the NCL	$\tilde{N}$	2.400
Poisson's ratio	ν	0.300
Initial void ratio	ν_0	2.734

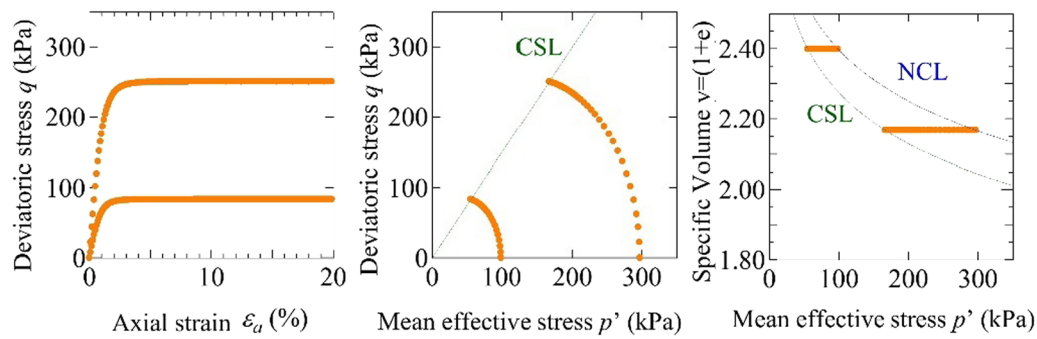


Fig. 1 Calculation results of the $\overline{\text{CU}}$ test using the parameters shown in Table 1

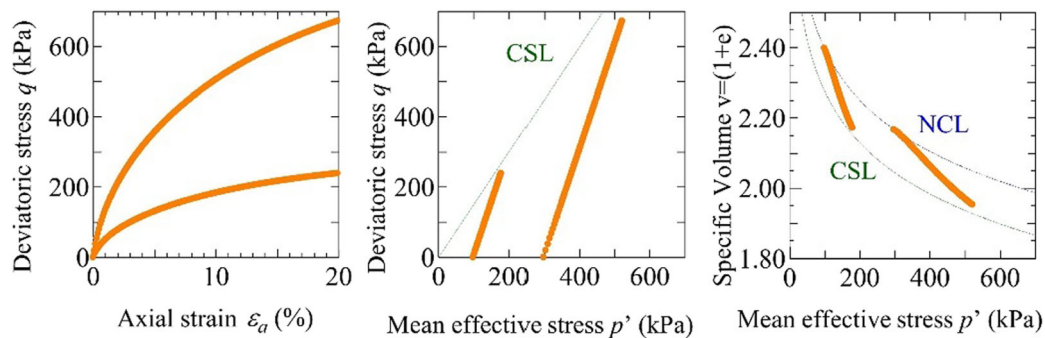


Fig. 2 Calculation results of the CD test using the parameters shown in Table 1

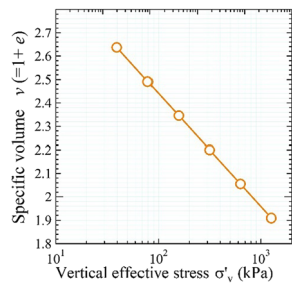


Fig. 3 Calculation results of the OED test using the parameters shown in Table 1

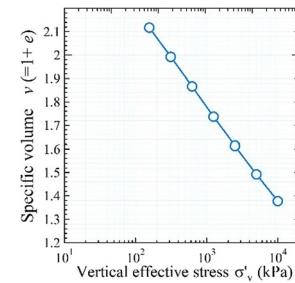


Fig. 5 OED test results of remolded Joetsu clay [22]

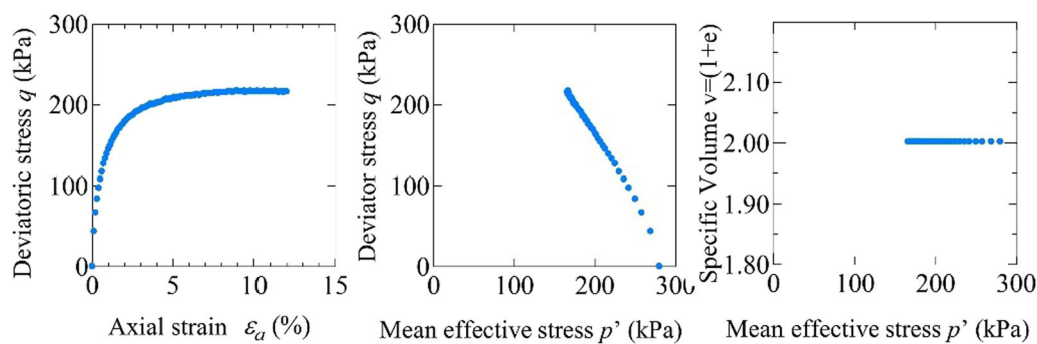


Fig. 4 $\overline{\text{CU}}$ test results of reconstituted Joetsu clay [22]

3 The algorithm for automatic parameter estimation

In this section, we describe an algorithm for automatically estimating the parameters of a constitutive model from the targeted results presented in Sect. 2, and we define an objective function that quantitatively evaluates the discrepancy between the targeted results and the calculation results. The algorithm minimizes the objective function to estimate the optimal parameters. This approach to solving the inverse problem is a direct approach [3, 16]. According to Knabe et al. [16], the direct approach is a suitable and robust algorithm for dealing with nonlinear models. We first define the objective function (error function) and then describe the DMS-PSO algorithm for minimizing the error function, including a flowchart of the analysis process. This inverse problem is also classified as a constrained optimization problem because it adopts search ranges of the parameters. In many previous studies, the effect of the parameter search ranges was not considered. In this section, we also show how to set the search range based on the physical property test results.

3.1 Definition of the objective function (error function)

Defining the objective function in this problem is crucial, and it is obvious that the closer the estimation results are to the targeted results, the better the parameter evaluation. Therefore, here, the objective function is defined by quantifying the discrepancy between the targeted results and the estimated results. In previous studies, some objective functions were proposed. For some studies that defined objective functions using the first and second invariants of the stress and strain tensors [4@-6], the errors between the deviator stress q and void ratio e in drained triaxial tests and between q and the excess pore water pressure Δu in drained triaxial tests were quantified by the least-squares method [7@-8]. With the horizontal axis representing time and the vertical axis representing the measured quantity, the measurement errors of t and $t + \Delta t$ were evaluated as the area [7]. Another study evaluated the error in strain [39]. The objective function was defined using weighted least-squares methods for volumetric strain ε_v in consolidation tests, and for q , ε_v , shear strain ε_s in drained

triaxial tests [16]. One study calculated the mean relative error using q , mean effective stress p' , ε_v , Δu , and e [43]. Another study estimated the squared error of q and the absolute value of the error considering the number of cycles until liquefaction [18]. Error has also been quantified by the squared error between experimental and calculation results of the stress–strain relationship [42]. Yet another study quantified the errors in the standard deviation σ_c^2 and contact force σ_f^2 of the mean stress p , q , and e by the least-squares method [37].

This study defined an objective function in v - p' - q space. v is the specific volume, p' is the mean effective stress, and q is the deviator stress. The modified Cam-Clay model is able to describe the behavior of remolded normal consolidated soil and coincides with the Roscoe surface, which exists in v - p' - q space as the state boundary surface, independent of the test type (such as drained or undrained shearing or consolidated tests). Therefore, the objective function defined in v - p' - q space is expected to be compatible with the modified Cam-Clay model and can quantify the discrepancy regardless of the test conditions. To easily evaluate the optimum values, the relative maximum and minimum values of v , p' , and q were used as variables for the objective function, as shown in Eqs. (1–3).

$$R_k^{p'} = \frac{p' - p'_{kTmin}}{p'_{kTmax} - p'_{kTmin}} \quad (1)$$

$$R_k^q = \frac{q - q_{kTmin}}{q_{kTmax} - q_{kTmin}} \quad (2)$$

$$R_k^v = \frac{v - v_{kTmin}}{v_{kTmax} - v_{kTmin}} \quad (3)$$

where the index k represents the k -th test. For example, when triaxial compression testing was conducted with confining pressures of 100 kPa and 300 kPa, the first test ($k = 1$) considered 100 kPa, and the second test ($k = 2$) considered 300 kPa. p'_{kTmax} , p'_{kTmin} , q_{kTmax} , q_{kTmin} , v_{kTmax} , and v_{kTmin} are the maximum and minimum p' , q , and v in the k -th targeted results. A space is defined for each test, and it is expected that the estimation results can be compared with the targeted results on the same scale, even though the experiments were conducted under different conditions.

The objective function is defined below:

$$\text{Error}(\mathbf{p}) = \frac{\sum_k^K \sum_j^J \sqrt{\left(R_{j\text{EST},k}^{p'}(\mathbf{p}) - R_{j\text{TGT},k}^{p'}\right)^2 + \left(R_{j\text{EST},k}^q(\mathbf{p}) - R_{j\text{TGT},k}^q\right)^2 + \left(R_{j\text{EST},k}^v(\mathbf{p}) - R_{j\text{TGT},k}^v\right)^2}}{J \cdot K} \quad (4)$$

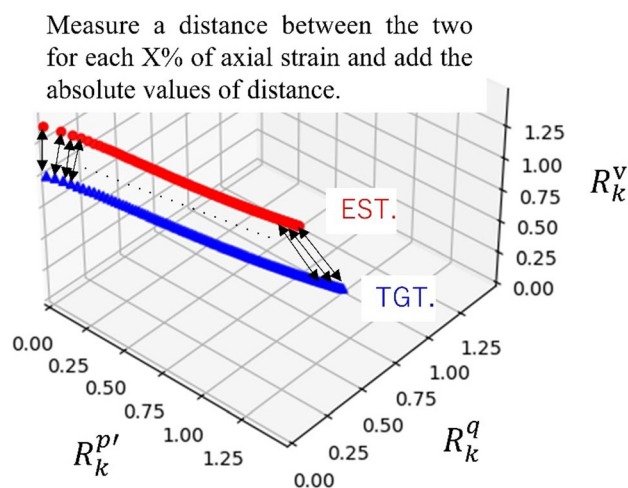


Fig. 6 Comparison of targeted results (TGT.) and estimated results (EST.)

where $\mathbf{p}$ is a vector whose components are the parameters of the constitutive model, depending on the constitutive model and the parameters to be estimated. Equation (8) measures the distance between the targeted results and the estimated results in R_k^p – R_k^q – R_k^v space. In other words, it quantifies the discrepancy between the targeted results and the estimated results. Figure 6 shows plots of an example of CD test results in R_k^p – R_k^q – R_k^v space. $R_{j\text{EST},k}^p(\mathbf{p})$, $R_{j\text{EST},k}^q(\mathbf{p})$, $R_{j\text{EST},k}^v(\mathbf{p})$, $R_{j\text{TGT},k}^p$, $R_{j\text{TGT},k}^q$, and $R_{j\text{TGT},k}^v$ are the j -th targeted and estimated results in the R_k^p – R_k^q – R_k^v space. J represents the number of data points from a test. When the triaxial test was conducted until shear strain ε_{fin} (%), the targeted and estimated results were compared for each shear strain X , $J = \varepsilon_{\text{fin}}/X$. In this study, $X = 0.1\%$.

For OED tests, because this study considered an incremental loading consolidation test, J represents the number of loading steps. In the OED test, only the difference between $R_{j\text{calc},k}^v$ and $R_{j\text{exp},k}^v$ can be used to quantify the discrepancy.

K is the number of tests conducted. When two kinds of tests, CU tests and OED tests, were performed, $K = 3$.

The difference between the j -th targeted and estimated results in R_k^p – R_k^q – R_k^v space for the k -th test was calculated. According to the numerator of Eq. (8), this difference is the distance between the targeted and estimated results in— R_k^p – R_k^q – R_k^v space. The j differences were summed and then divided by $J \cdot K$ to obtain the objective function. Since $J \cdot K$ indicates the total number of data points, dividing $J \cdot K$ enables the objective function to quantify the discrepancy on the same scale regardless of the number of data points and tests. The smaller the $\text{Error}(\mathbf{p})$ is, the better the evaluation of a particle. When $\text{Error}(\mathbf{p}) = 0$, the estimated results coincide with the targeted results.

3.2 DMS-PSO

PSO [15] is an algorithm inspired by fish or bird swarms. In PSO, all particles with position and velocity information in a multidimensional function space search for positions where an objective function is minimized, sharing their information with the whole swarm. This study adopted the DMS-PSO algorithm proposed by Liang and Suganthan [19]. Unlike in PSO, where all particles search as one swarm for their optimum positions, in DMS-PSO, N particles are divided into L swarms (expressed as L islands in this study) of M particles each to search for their optimum positions ($N = L \times M$). Particles must be equally divided among the islands. After updating the particle positions 15 times, the particles were randomly reallocated to a new set of islands. Figure 7 shows an example of updating the particle positions and reallocating the particles to the various islands. The updating equations for the velocity $\mathbf{v}_n^t$ and position $\mathbf{x}_n^t$ of the n -th particle at generation t of the DMS-PSO are as follows:

$$\mathbf{v}_n^{t+1} = w\mathbf{v}_n^t + c_1r_1(\mathbf{pbest}_n^t - \mathbf{x}_n^t) + c_2r_2(\mathbf{lbest}_l^t - \mathbf{x}_n^t) \quad n = 1, 2, \dots, N, l = 1, 2, \dots, L \quad (5)$$

$$\mathbf{x}_n^{t+1} = \mathbf{x}_n^t + \mathbf{v}_n^{t+1} \quad (6)$$

Equation (5) shows the updating equation of the velocity, and Eq. (6) shows the updating equation of the position. $\mathbf{pbest}_n^t$ indicates the best particle position, where an objective function obtains the lowest value, among all considered positions of the n -th particle. In other words, $\mathbf{pbest}_n^t$ indicates the parameter when the estimation results of the n -th particle are the closest to the targeted results until generation t . $\mathbf{lbest}_l^t$ indicates the best position of a particle belonging to the l -th island, where an objective function obtains the lowest value among all previous positions on the l -th island. After the particle positions are updated several times via optimization calculations, the particles are reallocated to the islands to share the information among all the islands. This process avoids the problem of falling into a local solution, which is a disadvantage of original PSO.

c_1 , c_2 and w in Eq. (1) are the PSO parameters. The larger the c_1 is, the easier the particle moves forward to $\mathbf{pbest}_n^t$. Therefore, a large c_1 means that the position information of the particle is prioritized. The larger the c_2 is, the easier the particle moves forward to $\mathbf{lbest}_l^t$. Therefore, a large c_2 means that the position information of the island is prioritized. w is the inertia parameter. The larger the w is, the higher the particle velocity. Therefore, when w is small, although the particles can search for the best position within a wide range, the particles have difficulty converging. On the other hand, when w is small, the

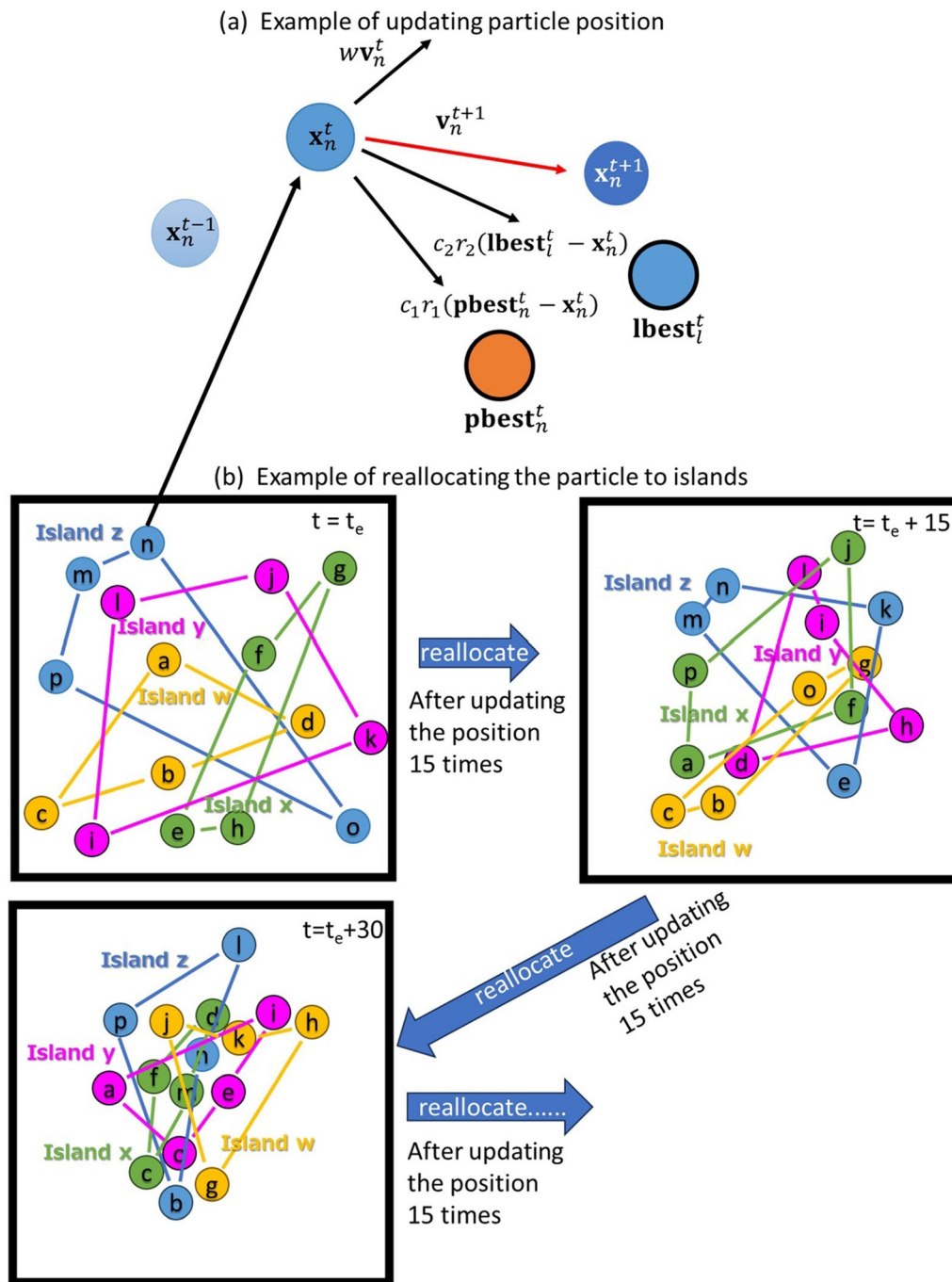


Fig. 7 Examples of **a** updating the particle positions and **b** reallocating the particles to the islands (the particles are labeled a through p, and the number of islands is four); the particles with the same color belong to the same island

particles can search carefully in a local area; however, they easily fall into an incorrect local solution in this case because the particle speed is low. c_1 and c_2 are usually set to values close to 1, while w is set to 0.8 or 0.9 [6]. In this study, $c_1 = c_2 = w = 0.9$. r_1 and r_2 are random numbers from 0 to 1.

The position of the particle is updated by substituting Eq. (1) into Eq. (2). Further details on the updating method are provided in STEP 9 below.

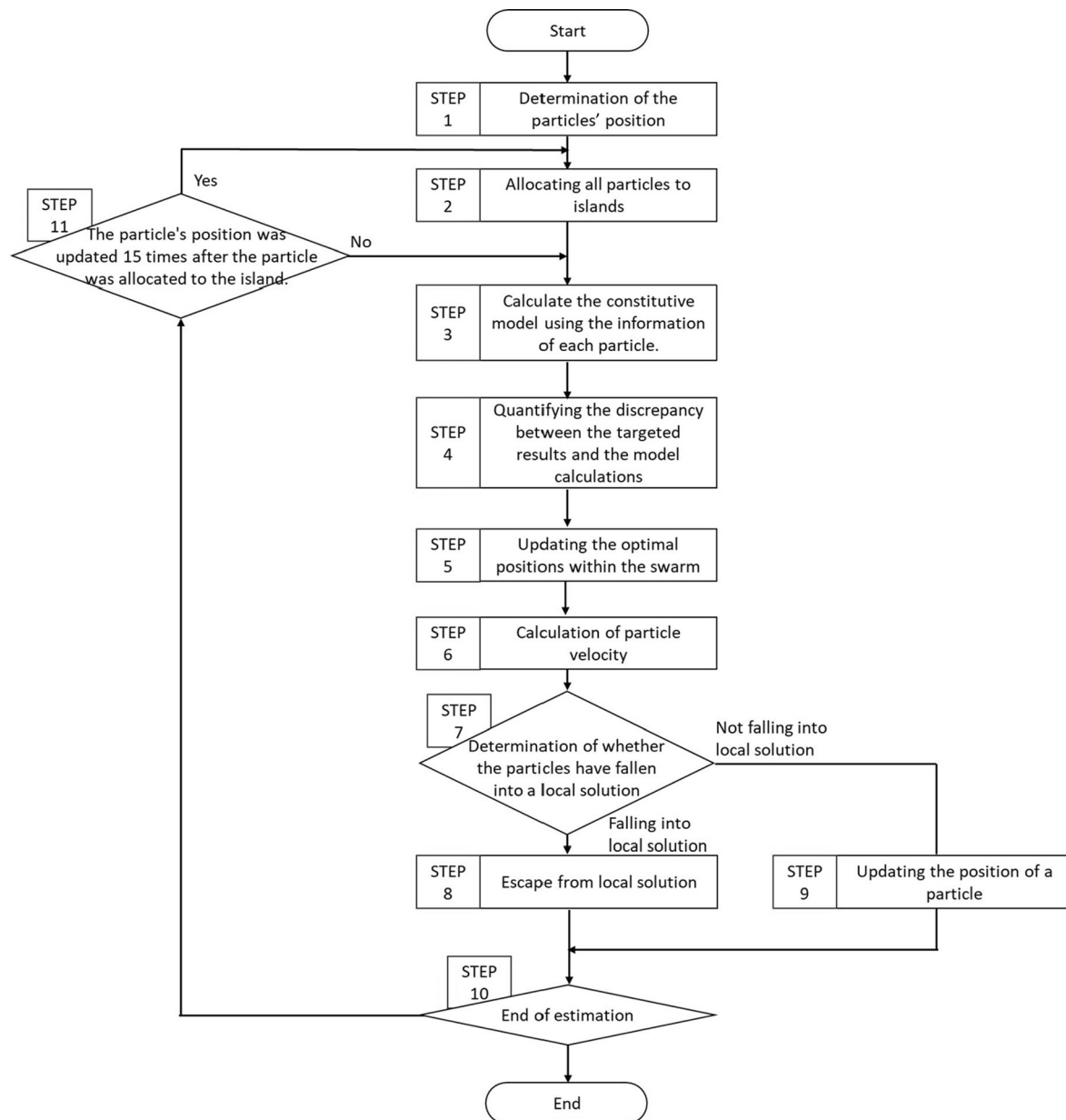


Fig. 8 Flowchart of automatic parameter estimation

3.3 Flowchart of automatic parameter estimation

Figure 8 shows the flowchart of automatic parameter estimation. Following this flowchart, we describe the procedure for parameter estimation.

STEP 1: The initial particle position $\mathbf{x}_n^1$ was determined using a uniform random number generated in the range where particles were able to exist, as shown in Sect. 3.5. Equation (7) shows the components of the particle position and velocity, which consists of the parameter constitutive model shown in Table 1.

$$\mathbf{x}_n^t = \begin{Bmatrix} \tilde{\lambda}_n^t \\ \tilde{\kappa}_n^t \\ \tilde{\mathbf{M}}_n^t \\ v_n^t \\ v_n^t \end{Bmatrix}, \mathbf{v}_i^t = \begin{Bmatrix} \Delta \tilde{\lambda}_n^t \\ \Delta \tilde{\kappa}_n^t \\ \Delta \tilde{\mathbf{M}}_n^t \\ \Delta v_n^t \\ \Delta v_n^t \end{Bmatrix} \quad (7)$$

The NCL intercept was obtained by the following equation:

$$v_0 = \tilde{\mathbf{N}} + \left(\tilde{\lambda} - \tilde{\kappa} \right) \ln \left(\frac{\tilde{\mathbf{M}}^2 - \eta_0^2}{\tilde{\mathbf{M}}^2} \right) - \tilde{\lambda} \ln p' \quad (8)$$

where $\eta_0 (= q_0/p'_0)$ is the initial stress ratio, q_0 is the initial deviatoric stress, and p'_0 is the initial mean effective stress.

v_0 is the initial specific volume. Initially, every component of $\mathbf{v}_n^t$ was set to zero.

STEP 2: The particles were allocated to islands. Particles were equally divided among the islands.

STEP3: The constitutive model calculation was conducted using the information of each particle position $\mathbf{x}_n^t$. After the calculations, results similar to those shown in Figs. 1, 2 and 3 were obtained.

STEP4: We calculated $Error(\mathbf{x}_n^t)$, which quantifies the difference between the targeted results and the calculation results of the constitutive model in $R_k^p-R_k^d-R_k^v$ space. For further details, please refer to Sect. 3.1.

STEP 5: The optimal positions within the swarm were updated. When $Error(\mathbf{x}_n^t) < Error(\mathbf{pbest}_n^t)$, $Error(\mathbf{x}_n^t) < Error(\mathbf{lbest}_n^t)$, and $Error(\mathbf{x}_n^t) < Error(\mathbf{gbest}^t)$ in terms of the number of generations in $t + 1$, $\mathbf{pbest}_n^t$, $\mathbf{lbest}_n^t$ and $\mathbf{gbest}^t$ are updated to $\mathbf{x}_n^t$, that is, $\mathbf{pbest}_n^t = \mathbf{x}_n^t$, $\mathbf{lbest}_n^{t+1} = \mathbf{x}_n^{t+1}$, and $\mathbf{gbest}^{t+1} = \mathbf{x}_n^{t+1}$. $\mathbf{gbest}^t$ indicates the position of the particle with the lowest value of the objective function among all previous positions. In other words, it is the estimated parameter that is closest to the targeted result among all known particle positions within the entire swarm.

STEP 6: The velocity of each particle $\mathbf{v}_n^{t+1}$ was obtained with Eq. (5).

STEP 7: The norm of the velocity $\|\mathbf{v}_i^t\|$ was used to determine whether a particle has fallen into a local solution. When the particles fall into a local solution, $\mathbf{pbest}_i^t = \mathbf{x}_i^t$, and $\mathbf{lbest}_j^t = \mathbf{x}_i^t$, which means that $\|\mathbf{v}_i^t\|$ is almost zero. Therefore, particles cannot be searched for further because they do not move. This is the reason why $\|\mathbf{v}_i^t\|$ is used to judge whether the particles are in a local solution. In this study, when $\|\mathbf{v}_i^t\|$ is less than ε for 80% of the particles, the particle was considered to have fallen into a local solution. ε was set to 0.0001 based on preliminary calculations.

STEP 8: To escape from the local solution, all the particles were relocated. In DMS-PSO, they are classified into

local search particles and global search particles when it is judged that the particles have fallen into a local solution. In this study, 50% of all the particles were classified as local search particles, and the remaining 50% were classified as global search particles. The global search particles were placed using the same method shown in STEP 1, i.e., the same method used for the initial arrangement, while local search particles were relocated near $\mathbf{gbest}^t$. The difference between the correct and estimated values decreases as $Error(\mathbf{x}_i^t)$ decreases, as shown in Fig. 9. Therefore, the particles were rearranged as in Eq. (9). Note that $\mathbf{v}_i^{t+1} = 0$ when rearranged.

$$\mathbf{x}_i^t = \{1 + rError(\mathbf{gbest}^t)\}\mathbf{gbest}^t \quad (9)$$

where r is a random number from -1 to 1.

STEP 9: Ordinarily, the particle position $\mathbf{x}_n^{t+1}$ is updated by substituting Eq. (5) for Eq. (6). If the component of $\mathbf{x}_n^{t+1}$ is outside the search range (details are given in Sect. 3.5), the particle is forced back into the search range. Assuming that the component of $\mathbf{x}_n^{t+1}$ is more than the maximum value of the range, the component is substituted for the maximum value. If the component of $\mathbf{x}_n^{t+1}$ is less than the minimum value of the range, that component is substituted for the minimum value. $\mathbf{v}_n^{t+1}$ is recalculated by $\mathbf{v}_n^{t+1} = \mathbf{x}_n^{t+1} - \mathbf{x}_n^t$.

STEP 10: When $Error(\mathbf{p}) < 1.0 \times 10^{-6}$, the estimation was finished. The parameters could be estimated to the fifth decimal place if $Error(\mathbf{p})$ was smaller than 1.0×10^{-6} from the preliminary calculations (shown in Fig. 9).

STEP 11: After updating the position 15 times, the particles were randomly relocated to the island, as shown in Fig. 7.

STEP 2 through 11 were repeated 1500 times.

3.4 Time complexity

In this study, we used a laptop computer with an AMD Ryzen Pro 6850 U with an 8-core CPU (2.7 GHz) and 16 GB of main memory. When estimating the parameters from a triaxial compression test and an oedometer test, it took 1.55 s with 800 particles and 0.76 s with 400 particles to proceed from STEP 2 to STEP 11. The computation time for STEP 3 (calculating the constitutive model using the information of each particle) was 1.44 s for 800 particles and 0.72 s for 400 particles. In other words, the time for optimization was 10% of the total computation time. In this study, the calculation was finished when the position of the particle was updated 1500 times if the value of the objective function was not less than 1.0×10^{-6} . When the number of particles was 800, the maximum computation time was $1.55 \text{ s} \times 1500 = 2325 \text{ s}$; when the number of

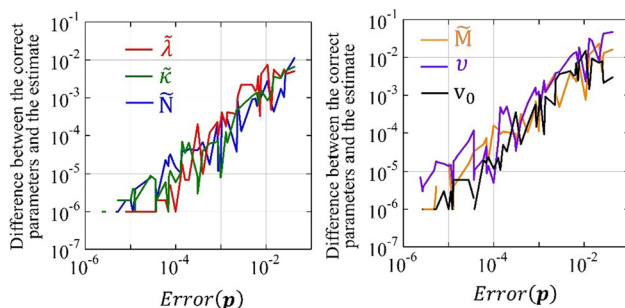


Fig. 9 Relationship between $Error(\mathbf{p})$ and the difference between the parameters used and the estimate

Table 2 Proposed equations for estimating $\tilde{\lambda}$ or $\tilde{\kappa}$

Equation	References
$\tilde{\lambda} = 0.4343 \times 0.007(w_L - 10)$	Skempton and Jones [29]
$\tilde{\lambda} = 0.4343 \times 0.0046(w_L - 9)$	Cozzolino [4]
$\tilde{\lambda} = 0.4343 \times 0.009(w_L - 10)$	Terzaghi and Peck [32]
$\tilde{\lambda} = 0.4343 \times 0.006(w_L - 9)$	Azzouz et al. [2]
$\tilde{\lambda} = 0.4343 \times 0.009(w_L - 8)$	Tuchida [34]
$\tilde{\lambda} = 0.4343 \times 0.009w_L$	Tsuchida [34]
$\tilde{\lambda} = 0.4343 \times 0.008(w_L - 12)$	Sridharan and Ngaraj [31]
$\tilde{\kappa} = 0.4343 \times (7 \times 10^{-6}w_L^2 + 0.0014w_L)$	Tiwari and Ajmera [33]
$\tilde{\kappa} = 0.4343 \times (9 \times 10^{-6}w_L^2 + 0.0002w_L)$	Tiwari and Ajmera [33])
$\tilde{\kappa} = 0.4343 \times (6 \times 10^{-6}I_p^2 + 0.0026I_p)$	Tiwari and Ajmera [33])
$\tilde{\kappa} = 0.4343 \times (2 \times 10^{-6}I_p^2 + 0.001I_p)$	Tiwari and Ajmera [33]
$\tilde{\kappa} = 0.0084(I_p - 4.6)$	Nakase et al. [23]

particles was 400, the maximum computation time was 1140 s. Therefore, the computational load is minimal.

3.5 How to set the solution search range

The liquid limit w_L or plastic index I_p can be used to set the range of parameters. Some equations that can estimate $\tilde{\lambda}$ and $\tilde{\kappa}$ have been proposed. These equations are summarized in Table 2. By substituting w_L and I_p into these equations, several $\tilde{\lambda}$ and $\tilde{\kappa}$ values were obtained. The maximum and minimum values among the obtained values were used as the search range. The critical state index $\tilde{M}$ can be estimated to some extent if the soil reaches the critical state in a shearing test. In this study, assuming that $\tilde{M}$ could be estimated within ± 0.025 from the test results, the search range was set. When the values of w_L , I_p and $\tilde{M}$ are not available, the ranges of $\tilde{\lambda}$, $\tilde{\kappa}$, and $\tilde{M}$ were assumed to be 0.030–0.400, 0.001–0.070, and 1.000–2.000, respectively. In this study, the range of Poisson's ratio ν was set to 0.01–0.49. For the specific volume v , the error was set to ± 0.025 to consider the possibility of a slight measurement error during the experiment.

4 The validation of the parameters

In this section, whether the constitutive model parameters can be correctly estimated from the results computed by the model, which were regarded as experimental results, and the validity of the DMS-PSO algorithm were examined. Some DMS-PSO parameters were given several values to investigate the effect of these parameters on the ability to search for a solution. To evaluate the ability to search for a solution, this study attempted to estimate the parameters 100 times and investigated the probability of estimating the correct parameters. In Sect. 4.1, we investigate the effect of the numbers of particles N and islands L on the ability. In Sect. 4.2, we investigate the effect of the search range and the number of tests with N and L optimized in Sect. 4.1

Narrowing the solution search range in PSO significantly is expected to improve the calculation stability and reduce the computational cost. Therefore, the effect of the search range on the estimation was investigated. To narrow the search range, we focused on the liquid limit and plastic index. The parameter search range is shown in Table 3. In this study, $w_L = 97.4\%$ and $I_p = 54.1$ [21]; these values were substituted for the equations shown in Table 2, and the ranges of $\tilde{\lambda}$ and $\tilde{\kappa}$ were set as shown in Table 3. The range of $\tilde{M}$ was determined by following the method described in Sect. 3.5. Note that $\tilde{M}$ of the alluvial clay is 1.5 [21]. In Sect. 4.1, a narrow range was used for the investigation.

4.1 The effect of the numbers of particles N and islands L

In this section, we investigate the effects of the number of particles N , the number of islands L , the number of particles per island M and the targeted mechanical test on the estimation capability. The experiments were $\overline{CU}$ and CD tests with a confining pressure of 300 kPa, as shown in Figs. 1 and 2. In this section, the narrow range shown in Table 3 is used for the investigation. Table 4 shows the N , L , M and targeted tests. The table also shows the results of the verification. Each case was evaluated by the number of times N_s that $Error(\mathbf{gbest}^t) < 1.0 \times 10^{-6}$ in 100 estimations, as well as the mean value (Av.) of the number of generations t until $Error(\mathbf{gbest}^t) < 1.0 \times 10^{-6}$ and the standard deviation (Std.) of t . For cases where $N_s \neq 100$,

Table 3 Range of the search for the solution

	$\tilde{\lambda}$	$\tilde{\kappa}$	$\tilde{M}$	ν	v
Narrow	0.177–0.380	0.026–0.069	1.45–1.55	0.01–0.49	2.709–2.759
Wide	0.030–0.400	0.001–0.070	1.00–2.00	0.01–0.49	2.709–2.759

Table 4 Study cases and results

Case	N	L	M	Test	N _s	Av.	Std.
1	200	5	40	$\overline{CU}$	69	–	–
2	200	10	20	$\overline{CU}$	85	–	–
3	200	20	10	$\overline{CU}$	97	–	–
4	200	40	5	$\overline{CU}$	97	–	–
5	200	100	2	$\overline{CU}$	90	–	–
6	400	5	80	$\overline{CU}$	78	–	–
7	400	10	40	$\overline{CU}$	90	–	–
8	400	20	20	$\overline{CU}$	92	–	–
9	400	40	10	$\overline{CU}$	100	532.8	109.2
10	400	80	5	$\overline{CU}$	100	612.0	111.5
11	400	200	2	$\overline{CU}$	100	845.2	178.3
12	800	1	800	$\overline{CU}$	11	–	–
13	800	5	160	$\overline{CU}$	82	–	–
14	800	10	80	$\overline{CU}$	92	–	–
15	800	20	40	$\overline{CU}$	98	–	–
16	800	40	20	$\overline{CU}$	100	446.4	89.6
17	800	80	10	$\overline{CU}$	100	469.8	73.9
18	800	160	5	$\overline{CU}$	100	585.5	70.7
19	800	400	2	$\overline{CU}$	100	798.0	119.4
20	200	5	40	CD	75	–	–
21	200	10	20	CD	95	–	–
22	200	20	10	CD	98	–	–
23	200	40	5	CD	100	592.0	72.2
24	200	100	2	CD	100	758.9	77.2
25	400	5	80	CD	88	–	–
26	400	10	40	CD	97	–	–
27	400	20	20	CD	100	418.6	53.2
28	400	40	10	CD	100	455.2	42.7
29	400	80	5	CD	100	541.2	42.6
30	400	200	2	CD	100	704.4	47.9
31	800	1	800	CD	25	–	–
32	800	5	160	CD	96	–	–
33	800	10	80	CD	99	–	–
34	800	20	40	CD	100	342.3	29.0
35	800	40	20	CD	100	384.1	39.6
36	800	80	10	CD	100	436.8	23.1
37	800	160	5	CD	100	507.1	29.1
38	800	400	2	CD	100	669.4	31.4

because the search for the correct value sometimes fails, that is, $Error(\mathbf{gbest}^t)$ sometimes does not become less than 1.0×10^{-6} , Av. and Std cannot be calculated. Note that Case 12 and Case 31, where $L = 1$, correspond to the original PSO.

The greater the number of particles, the greater the probability that the parameters were successfully estimated. In some cases, the probability of successful estimation was 100%, i.e., $N_s = 100$, when the number of particles was 400 or 800. On the other hand, original PSO was less successful, with a probability of successful estimation of 11% for Case 12 and 25% for Case 31. DMS-PSO had a much greater estimation ability than PSO. The solution was successfully searched when the number of particles was greater than 400, and the number of islands was greater than 40. However, as the number of islands increased, Av. increased. In particular, for the $\overline{CU}$ test, Std. also increased. For example, when the number of particles N was 400 and the number of islands L was 40 or 200, both Av. and Std. were 0.6 times smaller for 40 than for 200. However, the optimal number of islands should be considered since there was a risk of estimation failure if the number of islands was less than 20. In addition, parameter estimation tended to be easier with the CD test than with the $\overline{CU}$ test. With the same numbers of particles and islands, N_s was 0.9 times smaller and Std. was 0.2 times smaller for the CD test than for the $\overline{CU}$ test. The reason for this was explained using Figs. 1 and 2. In the CD tests in Fig. 2, the deviatoric stress q and specific volume v changed with increasing shear strain ε_s . On the other hand, since q hardly changed after ε_s reached 3%, as shown in Fig. 1, less information was obtained than in the CD test. In this study, the targeted and estimated results were compared for every 0.1% strain. These results suggested that it was necessary to obtain detailed data where the change was significant. Based on the results of this section, $N = 400$, $L = 40$, and $M = 10$ are used in the next section.

4.2 The effect of the search range and the number of tests

The number of particles $N = 400$ and the number of islands $L = 40$ were used based on the results from Sect. 4.1. For the targeted tests, it was assumed that $\overline{CU}$ and CD tests at confining pressures of 100 and 300 kPa or OED tests were conducted. If only one test was possible, it was assumed that only one $\overline{CU}$ or CD test at confining pressures of 100 and 300 kPa was conducted at a single coning pressure; if two tests were possible, $\overline{CU}$ or CD tests were conducted at both confining pressures, $\overline{CU}$ and CD tests were conducted at different confining pressures, or one triaxial compression test and an OED test were conducted. The influence of the search range was also examined using two kinds of parameter estimation ranges, narrow and wide, as shown in Table 3.

Table 5 shows the study cases and their results, with Cases 1 through 8 assuming that only one experiment was

Table 5 Study cases and results

Case	Test	N	L	Range	N _s	Av.	Std.
1	$\overline{CU}$ (100 kPa)	400	40	Narrow	100	493.6	75.6
2	$\overline{CU}$ (300 kPa)	400	40	Narrow	100	532.8	109.2
3	$\overline{CU}$ (100 kPa)	400	40	Wide	100	537.4	101.2
4	$\overline{CU}$ (300 kPa)	400	40	Wide	100	609.3	190.1
5	CD (100 kPa)	400	40	Narrow	100	591.7	96.0
6	CD (300 kPa)	400	40	Narrow	100	455.2	42.7
7	CD (100 kPa)	400	40	Wide	99	–	–
8	CD (300 kPa)	400	40	Wide	100	483.1	40.8
9	$\overline{CU}$ (100, 300 kPa)	400	40	Narrow	100	278.1	14.1
10	CD (100, 300 kPa)	400	40	Narrow	100	392.2	27.3
11	$\overline{CU}$ (100, 300 kPa)	400	40	Wide	100	304.0	16.5
12	CD (100, 300 kPa)	400	40	Wide	100	418.4	23.2
13	$\overline{CU}$ (100 kPa), CD (100 kPa)	400	40	Narrow	100	341.6	16.2
14	$\overline{CU}$ (300 kPa), CD (300 kPa)	400	40	Narrow	100	337.5	21.5
15	$\overline{CU}$ (100 kPa), CD (100 kPa)	400	40	Wide	100	375.5	20.6
16	$\overline{CU}$ (300 kPa), CD (300 kPa)	400	40	Wide	100	371.4	17.4
17	$\overline{CU}$ (100 kPa), OED	400	40	Narrow	100	265.8	13.0
18	$\overline{CU}$ (300 kPa), OED	400	40	Narrow	100	309.3	13.8
19	CD (100 kPa), OED	400	40	Narrow	100	335.6	18.8
20	CD (300 kPa), OED	400	40	Narrow	100	321.8	16.9
21	$\overline{CU}$ (100 kPa), OED	400	40	Wide	100	294.3	14.8
22	$\overline{CU}$ (300 kPa), OED	400	40	Wide	100	282.6	17.0
23	CD (100 kPa), OED	400	40	Wide	100	360.8	16.9
24	CD (300 kPa), OED	400	40	Wide	100	345.6	18.2

conducted. Cases 9–24 involved two experiments. Since all the cases except Case 7 always succeeded in estimating the parameters, the parameters could be estimated in either one

or two experiments. However, when two experiments were performed, Av. and Std. were smaller than when only one experiment was conducted. In particular, Std. was reduced to approximately one-fifth, showing a remarkable improvement in stability.

The narrower the parameter range, the smaller the Av. and Std. tended to be. Av. and Std. were approximately 10% lower for the narrow search range than for the wide search range. In other words, narrowing the range increased the speed and stability of the estimation. In Cases 9–24, the influence of the search range was smaller than that in Cases 1–8. In addition, when the range was wide, estimation failure was possible, as in Case 7. This result suggested that it was important to narrow the parameter estimation range and conduct two types of mechanical tests.

5 Validation of automatic parameter estimation using the experimental results

The performance of the DMS-PSO algorithm is evaluated in Sect. 4. However, it was not verified whether the same parameter values were obtained each time the experimental results were reproduced. The verification was performed by reproducing the experimental results shown in Figs. 4 and 5 using the DMS-PSO algorithm. The narrow estimation range of parameters was determined based on the Joetsu clay properties [22], $w_L = 72.5\%$ and $I_p = 42.8$ and the critical state constant $M \sim 1.25$, as shown in Table 6. The wide estimation range was the same as that in Table 3. Note that the initial specific volumes were different for the $\overline{CU}$ test and the OED test. Therefore, they were estimated separately from those in Sect. 4.

Table 7 shows the estimation results using the $\overline{CU}$ test, and Table 8 shows the estimation results using both the $\overline{CU}$ test and the OED test. Here, the obtained parameters were evaluated by the Av. and the coefficient of variation, C.V. (= Std./Av.). Using the C.V., the variation in each parameter was evaluated on the same scale. The Av. and C.V. of each parameter obtained are summarized in Figs. 10 and 11. Different parameters were obtained from the triaxial tests (Cases 1–4) and the triaxial compression and oedometer tests (Cases 5–8) because the experimental approaches were different. The results of Cases 1–4 were approximately the same, and those of Cases 5–8 were approximately the same. The objective function were also

Table 6 Range of the search for the solution

	$\tilde{\lambda}$	$\tilde{\kappa}$	$\tilde{M}$	ν	V ($\overline{CU}$)	ν (OED)
Narrow	0.127–0.283	0.020–0.060	1.20–1.30	0.01–0.49	1.977–2.027	2.091–2.141
Wide	0.030–0.400	0.001–0.070	1.00–2.00	0.01–0.49	1.977–2.027	2.091–2.141

Table 7 Parameter values obtained by the $\overline{CU}$ tests

		Case 1		Case 2		Case 3		Case 4	
N		400		400		800		800	
L		40		40		80		80	
Range		Narrow		Wide		Narrow		Wide	
Test		$\overline{CU}$		$\overline{CU}$		$\overline{CU}$		$\overline{CU}$	
		Av.	C.V. (%)	Av.	C.V. (%)	Av.	C.V. (%)	Av.	C.V. (%)
Objective function	U	0.040	0.000	0.040	0.000	0.040	0.000	0.042	0.000
Compression index	$\tilde{\lambda}$	0.223	0.545	0.223	0.498	0.223	0.498	0.199	0.239
Swelling index	$\tilde{\kappa}$	0.055	0.544	0.055	0.499	0.055	0.499	0.050	0.200
Critical state index	$\tilde{M}$	1.280	0.000	1.280	0.000	1.280	0.000	1.273	0.022
Intercept of the NCL	$\tilde{N}$	2.235	0.545	2.237	0.498	2.237	0.498	2.210	0.010
Poisson's ratio	ν	0.100	0.000	0.100	0.000	0.100	0.000	0.001	1.028
Initial void ratio ($\overline{CU}$)	v_0	2.001	0.545	2.003	0.498	2.003	0.498	1.977	0.000

Table 8 Parameter values obtained by the $\overline{CU}$ tests and the OED tests

		Case5		Case6		Case7		Case 8	
N		400		400		800		800	
L		40		40		80		80	
Range		Narrow		Wide		Narrow		Wide	
Test		$\overline{CU}$, OED		$\overline{CU}$, OED		$\overline{CU}$, OED		$\overline{CU}$, OED	
		Av.	C.V. (%)	Av.	C.V. (%)	Av.	C.V. (%)	Av.	C.V. (%)
Objective function	U	0.042	0.000	0.042	0.000	0.042	0.000	0.042	0.0000
Compression index	$\tilde{\lambda}$	0.199	0.230	0.199	0.230	0.199	0.201	0.199	0.225
Swelling index	$\tilde{\kappa}$	0.050	0.195	0.050	0.195	0.050	0.165	0.050	0.187
Critical state index	$\tilde{M}$	1.273	0.018	1.273	0.018	1.273	0.023	1.273	0.024
Intercept of the NCL	$\tilde{N}$	2.210	0.010	2.210	0.010	2.210	0.009	2.210	0.010
Poisson's ratio	ν	0.110	1.010	0.110	1.010	0.110	0.840	0.111	0.968
Initial void ratio ($\overline{CU}$)	v_0	1.977	0.000	1.977	0.000	1.977	0.000	1.977	0.000
Initial void ratio (OED)	v_0	2.116	0.000	2.116	0.000	2.116	0.000	2.116	0.000

greater for Cases 1–4 than for Cases 5–8 because of the difficulty in parameter estimation.

In all cases, there was no variation in the value of the objective function, but the parameters varied. It is implied that several sets of parameters can express the same experimental results. However, the C.V. was at most 1%, and the variation was minimal; that is, approximately the same parameter values were obtained from a few experimental results. The larger the number of particles and the narrower the estimation range were, the smaller the C.V. Additionally, the C.V. was reduced by approximately 10%.

This algorithm can estimate approximately the same parameter values, with slight variation among the 100 estimation attempts, but the variation in the obtained parameter values is discussed. When only one experiment,

in this case, a $\overline{CU}$ test, was conducted (Cases 1–4), the parameter variation was compared within each case, and $\tilde{\lambda}$, $\tilde{\kappa}$, $\tilde{N}$, and v_0 had approximately the same C.V. values, resulting in similar variations. However, $\tilde{M}$ and ν did not vary. On the other hand, with $\overline{CU}$ and OED tests (Cases 5–8), comparing cases where only the experiments conducted were different, i.e., Case 1 and Case 5, Case 2 and Case 6, Case 3 and Case 7 and Case 4 and Case 8, the C.V. for $\tilde{\lambda}$ and $\tilde{\kappa}$ was reduced to approximately 40%. For $\tilde{N}$ and ν , the C.V. was almost zero, but the variation in ν was the largest. The greater the number of particles and the narrower the search area were, the smaller the parameter variation. This means that narrowing the range of parameter estimation by

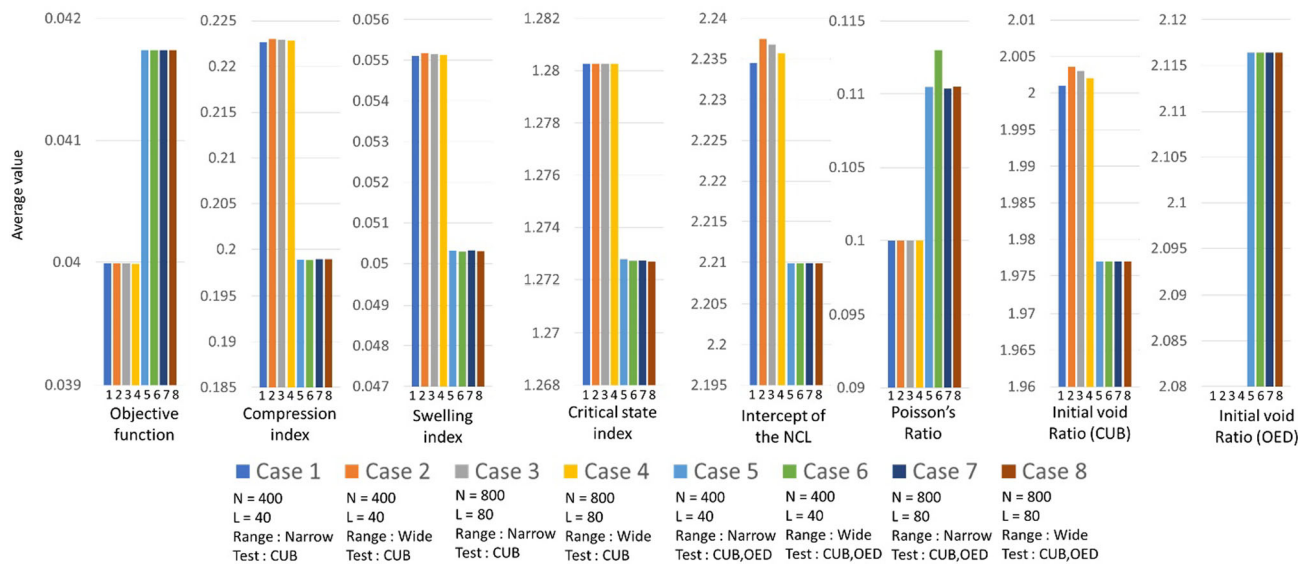


Fig. 10 Average parameter estimation results

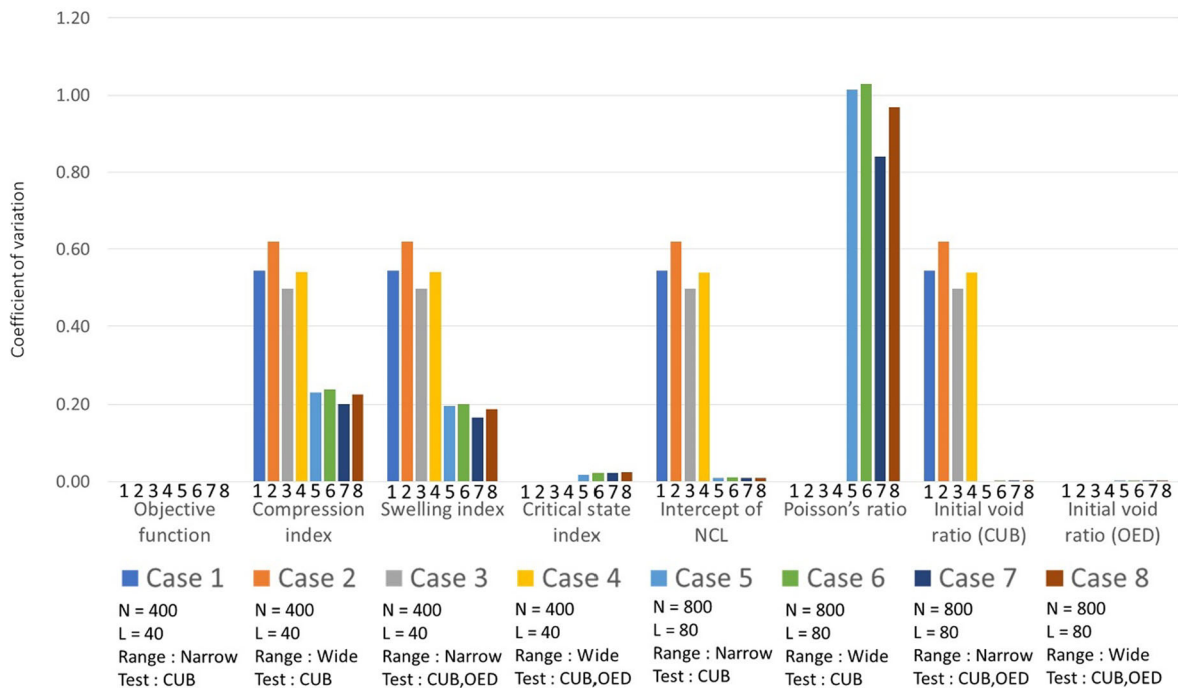


Fig. 11 C.V. of the parameter estimation results

physical property tests is an advantage for automatic parameter estimation.

Next, the reproduced results using the estimated parameters were studied. Figures 12 and 13 show the reproduced results of Case 7, where the parameters were estimated from the results of the two experiments. The reproduced results were in good agreement with the $\overline{CU}$ and OED test results. On the other hand, Fig. 14 shows the reproduced results of Case 3, where the parameters were estimated from one experimental result (the $\overline{CU}$ test). The

estimated results reproduced the $\overline{CU}$ result. However, the OED test could not be reproduced well by the estimated parameters, as shown in Fig. 15. These findings indicate that it is better to estimate parameters from two experiments than from one experiment.

The results suggest that the parameters can be identified accurately enough with at least two test results. However, even in the case of a single triaxial compression test, two sets of data can be obtained by using the isotropic consolidation process prior to shear. Therefore, a single triaxial

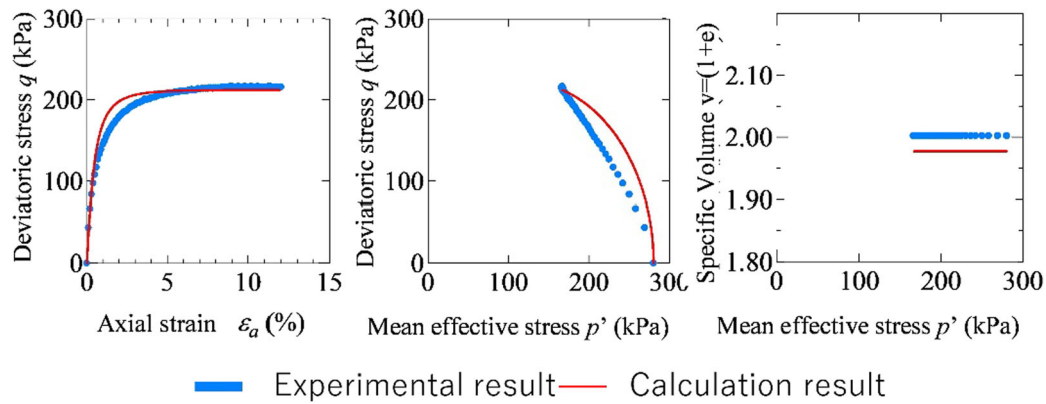


Fig. 12 Reproduced results of the $\overline{\text{CU}}$ test with the Case 7 parameters

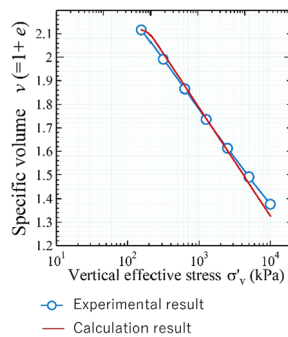


Fig. 13 Reproduced results of the OED test with the Case 7 parameters

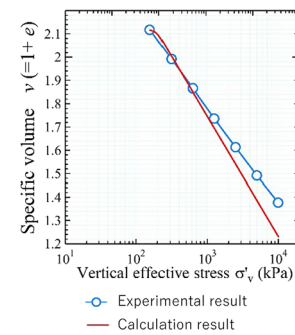


Fig. 15 Calculation results of the OED test using the parameters of Case 3

compression test may be used to estimate parameters with good accuracy.

6 Conclusions

In this study, the DMS-PSO algorithm, which is an improvement over the original PSO, was applied to automatically estimate the parameters of a modified Cam-Clay model. To examine the validity of the DMS-PSO

algorithm, we examined whether constitutive model parameters could be correctly estimated from calculation results (Consideration (1)) and whether the DMS-PSO algorithm could consistently obtain the same parameter values from experimental results (Consideration (2)). Consideration (1) was used to confirm whether the parameters used were estimated correctly by the DMS-PSO algorithm by estimating parameters from the calculation results obtained by the constitutive model as experimental results. In other words, for Consideration (1), the

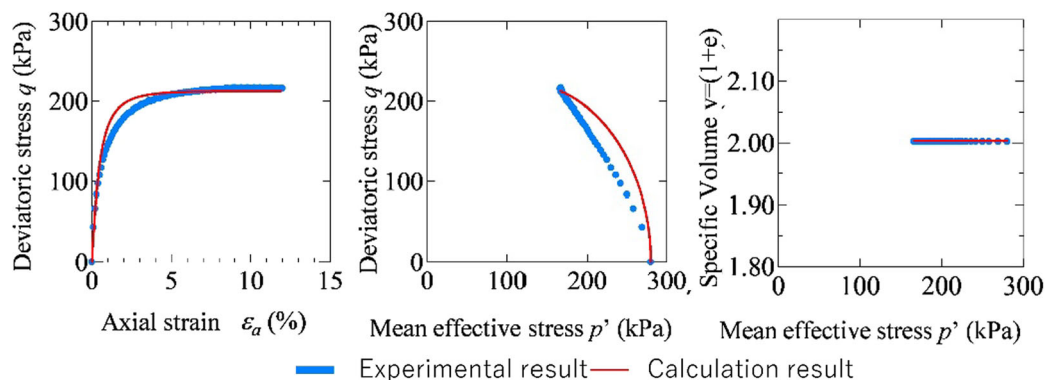


Fig. 14 Reproduced results of the $\overline{\text{CU}}$ test with the Case 3 parameters

performance of the DMS-PSO algorithm was examined. According to Consideration (2), the parameters could be automatically estimated from the experimental results via the DMS-PSO algorithm.

Regarding Consideration (1), the search was always successful if the number of particles was more than 400 and the number of islands was more than 40. Ordinary PSO was less successful, with a probability of successful estimation of 11% for Case 12 and 25% for Case 31. Therefore, the estimation capability was clearly improved by using DMS-PSO compared to original PSO. As the number of islands increased, N_s increased. In particular, for the $\overline{CU}$ test, Std. also increased. For example, when the number of particles N was 400 and the number of islands L was 40 or 200, both Av. and Std. were 0.6 times smaller for 40 than for 200. However, the optimal number of islands should be considered since there was a risk of estimation failure if the number of islands was less than 20. In addition, parameter estimation tended to be easier with the CD test than with the $\overline{CU}$ test. With the same numbers of particles and islands, N_s was 0.9 times smaller and Std. was 0.2 times smaller for the CD test than for the $\overline{CU}$ test. The Av. and Std. tended to be smaller when the search area was narrower. Av. and Std. were approximately 10% lower for the narrow search range than for the wide search range. Moreover, compared to when one experiment was conducted, when two experiments were conducted, the Av. and Std. were smaller. In particular, Std. was reduced to approximately one-fifth, showing a remarkable improvement in stability.

This indicated that the stability of the search for a solution was improved by conducting two experiments. Regarding Consideration (1), the effect of the search range was small when two experiments were conducted.

Regarding Consideration (2), parameters that reproduce the experimental results well were able to be automatically estimated from the experimental results. Since the C.V. was at most 1%, and the variation was minimal, this algorithm was able to estimate nearly the same parameter values among the 100 estimation attempts. The C.V. of the obtained parameters was decreased by 10% by narrowing the search range of the physical properties. Considering not only triaxial tests, but also consolidation tests, $\tilde{\lambda}$ and $\tilde{\kappa}$ were reduced to approximately 40%. For $\tilde{N}$ and v , the C.V. was almost zero, but the C.V. of v was 1.0%. In addition, the parameters estimated from one experiment were not able to reproduce other experiments. Therefore, parameters should be estimated by data from at least 2 mechanical experiments.

Although the modified Cam-Clay model was used in this study, the soil behavior that this model can reproduce is limited. Models that can represent the behavior of naturally

deposited clay, sandy soil, and improved soil, which the modified Cam-Clay model cannot describe, have been proposed (e.g., the SYS Cam-Clay model [1]). In the future, we would like to attempt to identify the parameters of these models.

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